Quasilinear diffusion as a result of modulational instability in the pulsar plasma

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Quasilinear diffusion due to modulational instability is considered in this paper. Interaction between the high-frequency, nearly transverse O mode (or the transverse X mode) and the low-frequency, nearly longitudinal L-O mode in a pulsar magnetospheric pair plasma can lead to modulational instability. The low-frequency L-O mode is superluminal, which is not subjected to usual Landau damping, and it is possible that excess wave energy is stored in this superluminal mode. The superluminal low-frequency L-O mode can dissipate in a way similar to the process of Langmuir wave collapse, that is, it cascades from the long- to short-wavelength regimes. When the phase speed becomes less than c, the waves can be damped through various resonances. We consider, in particular, damping through cyclotron resonance, which can lead to particle acceleration. The energetic beam particles, which have a very small spread initially, can develop a high-energy distribution tail, acquiring pitch angles through quasilinear diffusion. These particles can emit γ rays through synchrotron radiation, contributing to the observed pulsar high-energy emission.

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I. INTRODUCTION

It is widely believed that a pulsar magnetosphere is populated with dense electron-positron pair plasmas, which are produced above the polar cap through cascades by accelerated primary particles [1–4]. Such a plasma system, which includes a bulk secondary pair plasma and an energetic primary particle beam, is unstable, subjected to various plasma instabilities [5] (a recent review in Ref. [6]). There has been considerable interest in studying plasma processes in the pulsar magnetospheric plasma, mainly motivated by attempting to understand the pulsar emission mechanism [5,7–12]. Pulsars are observed to radiate coherent radio emission, which is believed to be originating from some type of plasma instability [6], and some pulsars also radiate high-energy emission [13], which may be due to synchrotron or cyclotron radiation from electron-positron cascades as in the polar gap [4,14] or outer gap models [15]. It has recently been shown that some plasma processes may also contribute to pulsar high-energy emission [12,16].

There were extensive discussions on possible modes in a relativistic pair plasma in a strong magnetic field [5,7,9,10,17]. Pulsars are believed to have very strong magnetic fields with typical strength ranging from 10^8 to 10^{13} G. Electrons or positrons moving in such strong magnetic fields radiate away rapidly their perpendicular energy and they move along the field lines one dimensionally. In practice, one may derive plasma dispersion by assuming zero perpendicular momentum $p_\perp=0$, but allowing transition to the first Landau level [8]. Further, since the bulk pair plasma consists of electrons and positrons, one may assume quasineutrality with charge symmetry, which allows the dispersion relations to be simplified considerably. In these approximations, one can obtain three distinct modes for oblique propagation (with respect to the field line direction). (1) The L-O mode with the polarization vector being in the **k**-**B** plane, which is superluminal at low frequency and high frequency regions; (2) the low frequency Alfven mode, which can be heavily damped if its phase speed is less than the bulk plasma speed; and (3) the X mode with the polarization vector being perpendicular to the **k**-**B** plane, which is subluminal at low frequency and is purely transverse. In the high-frequency region the L-O mode becomes transverse, called the O mode.

Since the low-frequency L-O mode is superluminal and not subject to Landau damping, even for a very modest efficient production mechanism there can be excess low-frequency L-O mode waves. Since the low-frequency L-O mode is mainly longitudinal in the superluminal region, it cannot escape from pulsar magnetospheres without conversion to high-frequency transverse waves. Nonlinear interaction between the low-frequency L-O mode and the high-frequency transverse O mode (or X mode) was recently studied in Refs. [18,11]. Assuming there preexists high-frequency O-mode (or X-mode) waves, Ref. [11] considered modulational instability of the O mode due to nonresonant interaction. Through nonresonant interaction, in which neither Cerenkov nor cyclotron resonance condition can be satisfied, the low-frequency superluminal L-O mode is converted to an unstable low-frequency beat wave with an oblique propagation angle and at the same time particles acquire a pitch angle emitting synchrotron radiation in X or γ rays [12].

In this paper, we discuss modulational instability of the low-frequency superluminal L-O mode by the beat of two high-frequency transverse waves and consider the subsequent quasilinear diffusion due to the resonant interaction of plasma particles and the unstable wave. Specifically we consider quasilinear diffusion as a result of cyclotron resonance by very energetic particles, such as, those in the primary beam. We assume that the two high-frequency transverse waves can be generated by a certain type of plasma instability.

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II. MODULATIONAL INSTABILITY

Modulational instability of the superluminal L-O mode by beat of two transverse waves was recently discussed in Ref. [11]. The transverse waves are referred to those high-frequency O mode or X mode waves (with frequency being much higher than ω_p and the dispersion is close to the vacuum case, $n=1$), which are assumed to be generated by some plasma instability, e.g., anomalous cyclotron resonance [19]. For two high-frequency transverse waves with frequencies ω^t and ω'^t and wave vectors $\mathbf{k}^t$ and $\mathbf{k}'^t$, it is possible to have a beat wave with a frequency $\Delta\omega=\omega^t-\omega'^t$, which has a longitudinal, superluminal component with the parallel phase speed given by

$$\frac{\Delta\omega}{|\Delta k_\parallel|}>c, \tag{2.1}$$

where $\Delta k_\parallel=k_\parallel^t-k'^t_\parallel$ and we assume $\omega^t>\omega'^t$. The beat of the two transverse waves can induce density modulation with $|\mathbf{k}_\perp^t-\mathbf{k}'^t_\perp|>|k_\parallel^t-k'^t_\parallel|$, leading to modulational instability of the low-frequency superluminal L-O mode with frequency $\omega_\mathrm{L}\ll\omega^t,\ \omega'^t$. For nearly parallel propagation, the low-frequency L-O mode is almost electrostatic [8]. The condition for instability requires that the beat frequency be small [11]

$$\frac{k_z c}{\Delta\omega}a_c>\mathcal{K}_0^2\left(\frac{k_\perp c}{\omega_p}\right)^2, \tag{2.2}$$

where $k_\perp$ is the perpendicular (to the magnetic field) component of the wave vector, $\mathcal{K}_0=k_0^l c/2\omega_p$, k_0^l is the wave number of electrostatic waves generated from interaction between the two high-frequency waves, $\omega_p=(4\pi e^2 n/m_e)^{1/2}$ with $n=n_++n_-$ the total plasma density (electrons and positrons) is the plasma frequency, $a_c=(\omega_p\Omega_e/\gamma_p\omega^{t2})|\boldsymbol{\mathcal{E}}_\perp^t|^2$, Ω_e is the cyclotron frequency, $\boldsymbol{\mathcal{E}}_\perp^t=e\mathbf{E}_\perp^t/(m_e c\omega_p)$ is the perpendicular component of the electric field of the high-frequency transverse wave. Thus, the energy of the superluminal L-O mode is converted to perturbations with the low frequency, $\omega^t-\omega'^t$. This instability has back reaction on the plasma particle distribution with quasilinear diffusion in the momentum space.

As shown in Refs. [11,12] the growth of modulational instability that leads to nonresonant interaction in the frequency region $|\omega-k_\parallel v_\parallel|\gamma/\Omega_e\ll1$. In the nonresonant approximation [20,21], plasma particles are subjected to nonresonant quasilinear diffusion (NQD) leading to transfer of particle's parallel energy to its perpendicular energy. The pitch angle $\psi=\arctan(p_\perp/p_\parallel)$ acquired by the plasma particles is given by [11,12]

$$\psi\approx\frac{B_\perp}{B}, \tag{2.3}$$

where $B_\perp$ is the magnetic field of the unstable waves, being perpendicular to the pulsar magnetic field B. Generally, plasma particles and the relevant unstable waves can have resonant interaction, which can result in resonant quasilinear diffusion (RQD) [22,23]. In the following discussion we specifically consider RQD resulting from modulational instability.

III. ABSORPTION OF WAVES

Assume that the unstable wave generated from modulational instability of the low-frequency superluminal L-O mode is subluminal with a frequency ω. The unstable wave interacts with plasma particles causing quasilinear diffusion. RQD requires a resonant interaction between the wave and particles. There are three types of resonance processes that can cause wave absorption: Landau, Landau drift, and cyclotron damping. We show that among them the cyclotron resonance is the most efficient.

A. Cyclotron damping

We first consider damping due to cyclotron absorption arising from normal cyclotron resonance (NCR), which can occur for waves with a superluminal parallel phase speed $\omega/k_\parallel>c$ [24]. Pulsar magnetic fields can be modeled as dipolar fields with the radius of field line curvature being approximated by $R_B\approx(4/3)(RR_\mathrm{LC})^{1/2}$ for the last open field lines, where R is the radial distance and R_LC is the radius of the light cylinder. Relativistic particles moving in a curved magnetic field have drift motion across the field lines with the drift velocity being

$$v_d=\frac{v_\parallel^2\gamma}{\Omega_e R_B}, \tag{3.1}$$

where $\Omega_e=eB/m_e c$ is the cyclotron frequency. The effect of field line curvature in generation of pulsar radio emission has been discussed by several authors, e.g., Ref. [19]. From Eq. (3.1), the curvature drift can be significant for ultrarelativistic particles.

Including the curvature drift, the NCR condition can be written as

$$\omega-k_\parallel v_\parallel-k_\perp v_d=\Omega_e/\gamma. \tag{3.2}$$

The condition can be satisfied for a subluminal wave with $\omega/k<c$ but $\omega/k_\parallel>c$. Throughout our discussion we consider the L-O or X mode in the quasitranverse approximation with dispersion $\omega=kc(1-\delta)$, where $|\delta|\ll1$ is a small correction to the vacuum dispersion. The condition for damping is $|\delta|<(k_\perp/k_\parallel-v_d/c)^2/2$, implying that waves with a nonzero propagation angle can be damped. The smaller the angle, the larger is the Lorentz factor required for particles to damp the waves through NCR. In the following, we consider NCR by energetic particles in the primary beam, which has a typical Lorentz factor of 10^7 for typical pulsars. The resonant frequency is

$$\omega_\mathrm{res}\approx\frac{\Omega_e}{\gamma_\mathrm{b,res}}\left(\frac{k_\perp}{k_\parallel}\right)^2, \tag{3.3}$$

where $\gamma_{b,res}$ is the Lorentz factor of the beam particles that are in cyclotron resonance and ω_{res} is the resonant frequency (hereafter we simply use ω). The damping rate is given by [23–25]

$$\Gamma^{NCR}=\pi\frac{\omega_b^2}{\omega}\frac{1}{\gamma_T}, \tag{3.4}$$

where ω_b is the plasma frequency of the beam, γ_T is the beam spread.

B. Landau damping

For Landau damping we have

$$\omega-k_\parallel v_\parallel=0. \tag{3.5}$$

Since particles in the fast beam can have a very large Lorentz factor and therefore, in general, as for Cerenkov instability the kinetic approximation is not valid for Landau damping [26] (more recently, cf. Ref. [10]). For particles with $\gamma=\gamma_0+\Delta\gamma$, where γ_0 is the Lorentz factor that satisfies the resonance $\omega_{res}-k_\parallel c(1-1/2\gamma_0^2)=0$, $\Delta\gamma\ll\gamma_0$ is the spread, we have $\omega-k_\parallel v_\parallel\approx\omega_{res}\Delta\gamma/\gamma_0^3$. For a nearly parallel propagation wave, the condition (3.5) can be satisfied only for very energetic particles. Since the Landau (temporal) damping rate is $\Gamma_D\propto\gamma^3$, we have $|\omega-k_\parallel v_\parallel|\ll\Gamma_D$ for a large γ_0. Therefore, the kinetic approximation is not applicable.

Damping or instability associated with energetic particles must be in the hydrodynamic regime, i.e., the resonance width is much smaller than the relevant growth or damping rate. However, it can be shown that damping or instability in the hydrodynamic regime is small. Thus, Landau damping is not effective [26] and will not be considered further.

C. Landau-drift resonance

The usual Landau resonance condition (3.5) is modified by a drift term, that is,

$$\omega-k_\parallel v_\parallel-k_\perp v_d=0, \tag{3.6}$$

which we call the Landau-drift resonance. In the kinetic approximation the resonant condition can be written as

$$\delta\approx\frac{1}{2}\left(\frac{k_\perp}{k_\parallel}-\frac{v_d}{c}\right)^2. \tag{3.7}$$

Although the damping due to Landau-drift resonance can be important, the condition (3.7) can only be satisfied in a narrow parameter range. So, in the following discussion we consider absorption due to NCR only.

IV. RESONANT QUASILINEAR DIFFUSION DUE TO NCR

Let $f(p_\parallel,\psi)$ be the plasma distribution where ψ is the pitch angle defined by $\tan\psi=p_\perp/p_\parallel$ with $p_\parallel$ and $p_\perp$ being, respectively, the parallel and perpendicular momentum (relative to the magnetic field field direction). In the strong pulsar magnetic field we have the approximation $p_\perp/p_\parallel\equiv\tan\psi\approx\psi\ll1$, and the RQD equation is [23,27]

$$\frac{df(p_\parallel,\psi)}{dt}\approx\frac{1}{\psi}\frac{\partial}{\partial\psi}\left[\psi\left(D_{\psi\psi}\frac{\partial}{\partial\psi}+D_{\psi\parallel}\frac{\partial}{\partial p_\parallel}\right)f(p_\parallel,\psi)\right]+\frac{\partial}{\partial p_\parallel}\left[\left(D_{\parallel\psi}\frac{\partial}{\partial\psi}+D_{\parallel\parallel}\frac{\partial}{\partial p_\parallel}\right)f(p_\parallel,\psi)\right], \tag{4.1}$$

where the diffusion coefficient D is given by

$$\begin{bmatrix} D_{\psi\psi} \\ D_{\parallel\psi}=D_{\psi\parallel} \\ D_{\parallel\parallel}\end{bmatrix}=\sum_s\int\frac{d\mathbf{k}}{(2\pi)^3}w(s,\mathbf{k},\mathbf{p})N_{\mathbf{k}}\begin{bmatrix}(\Delta\psi)^2 \\ \Delta\psi\Delta p_\parallel \\ (\Delta p_\parallel)^2\end{bmatrix}. \tag{4.2}$$

$$w(s,\mathbf{k},\mathbf{p})=\frac{4\pi^2e^2}{\hbar\omega_k}|\mathbf{e}^*(\mathbf{k})\cdot\mathbf{V}(s,\mathbf{k},\mathbf{p})|^2\delta(\omega-k_\parallel v_\parallel-s\Omega_e/\gamma), \tag{4.3}$$

where

$$\mathbf{V}(s,\mathbf{k},\mathbf{p})=[v_\perp sJ_s(\varrho)/\varrho,-i\eta sv_\perp J_s'(\varrho),v_\parallel J_s(\varrho)],$$

$$\varrho=k_\perp v_\perp\gamma/\Omega_e,$$

η is the charge sign, $\Delta p_\parallel=\hbar k_\parallel$, $\Delta\psi=s\hbar\Omega_e m_e/p_\parallel p_\perp$, $\mathbf{e}(\mathbf{k})$ is the polarization vector, $w(s,\mathbf{k},\mathbf{p})$ is the probability for cyclotron emission, and $N_{\mathbf{k}}$ is the wave occupation number. The Landau damping corresponds to $s=0$, which is not considered here. Equation (4.1) is in the small pitch angle approximation $\psi\ll1$, which is relevant for pulsar magnetospheric plasmas. For NCR, we only need to consider $s=1$.

As shown in Refs. [18,11], the unstable wave generated through modulational instability has substantial transverse component. For convenience, we assume the polarization vector simply to be $\mathbf{e}=(0,i,0)$, we have $|\mathbf{e}^*\cdot\mathbf{V}|^2=(v_\perp^2/4)$. The three relevant components are

$$D_{\psi\psi}\approx\frac{r_e}{2m_ec\gamma^3}\hbar\Omega_e n_k, \tag{4.4}$$

$$D_{\parallel\psi}\approx\frac{r_e\psi}{2\gamma}\hbar\omega_k n_k, \tag{4.5}$$

$$D_{\parallel\parallel}=\frac{r_e}{4}m_ec\gamma\psi^2\frac{\omega_k}{\Omega_e}\hbar\omega_k n_k, \tag{4.6}$$

where $p_\parallel/m_ec\approx\gamma$ (in the pulsar frame the parallel momentum is always positive), $r_e=e^2/m_ec^2\approx10^{-13}$ cm is the classic electron radius, $n_k=\int N_{\mathbf{k}}(d\mathbf{k}_\perp/2\pi)$, with $\mathbf{k}=\mathbf{k}_{res}$ at the cyclotron resonance. For the parallel diffusion coefficient $D_{\parallel\parallel}$ we neglect the Cerenkov resonance, which is not considered here. Equation (4.5) has positive sign, implying acceleration. Lominadze *et al.* studied RQD due to anomalous cyclotron resonance and derived the parallel-perpendicular component of D, which has a minus sign, corresponding to deceleration, e.g., Eq. (4.5) in Ref. [23]. Application of anomalous cyclotron instability to pulsar emission was discussed by Machabeli and Usov [28].

The right-hand side of Eq. (4.1) describes stimulated processes in which the effectiveness is proportional to the wave occupation number $N_{\mathbf{k}}$. However, radiation reaction can also change the particle distribution. There are two ways to treat this effect, which are equivalent; one way is to consider the reaction as a spontaneous term, which is added in Eq. (4.1); another way is to treat it as an effective force, which can be included on the left-hand side. Here we adopt the latter, that is, the left-hand side of Eq. (4.1) can be written as

$$\frac{df}{dt}=\frac{\partial f}{\partial t}+\frac{\partial}{\partial \mathbf{p}}\cdot[(\mathbf{G}+\mathbf{F})f]$$
$$=\frac{1}{p_\parallel \psi}\frac{\partial}{\partial \psi}[\psi(G_\perp+F_\perp)f]+\frac{\partial}{\partial p_\parallel}[(G_\parallel+F_\parallel)f], \quad (4.7)$$

where $\mathbf{F}$ is the effective force due to radiation and damping, such as synchrotron radiation and damping, and $\mathbf{G}$ is the effective force induced by the magnetic field inhomogeneity [28]. For $p_\perp \neq 0$, we have [28,8]

$$G_\parallel=\beta_R\gamma\psi^2, \quad G_\perp=-\beta_R\gamma\psi, \quad (4.8)$$

where $\beta_R=mc^2/R_B$, R_B is the radius of field line curvature. The radiation reaction force due to synchrotron and curvature radiation is given by

$$F_\parallel=-\alpha\gamma^2\psi^2-\alpha_c\gamma^4, \quad (4.9)$$

$$F_\perp=-\alpha\psi(1+\gamma^2\psi^2), \quad (4.10)$$

where the second term in Eq. (4.9) is due to curvature radiation, $\alpha=2e^2\Omega_e^2/3c^2=2e^2/3\rho_L^2$, $\rho_L=c/\Omega_e$, $\alpha_c=2e^2/3R_B^2$. Since $\alpha/\alpha_c=(R_B/\rho_L)^2\ll 1$, curvature radiation is important only for very large γ. Since $G_\parallel \ll F_\parallel$, the terms with $G_\parallel$ can be neglected.

For convenience we introduce parallel momentum distribution $f_\parallel(p_\parallel)$ and pitch angle distribution $f_\perp(\psi)$, defined, respectively, by integration of $f(p_\parallel,\psi)$ over pitch angle and parallel momentum, i.e.,

$$f_\parallel(p_\parallel)=\int_0^{\pi/2}\psi f(p_\parallel,\psi)d\psi, \quad f_\perp(\psi)=\int_{-\infty}^{\infty}f(p_\parallel,\psi)dp_\parallel . \quad (4.11)$$

Integrating Eq. (4.1) over $p_\parallel$, we find

$$\frac{\partial f_\perp(\psi)}{\partial t}\approx-\frac{1}{p\psi}\frac{\partial}{\partial\psi}[\psi(G_\perp+F_\perp)f_\perp(\psi)]$$
$$+\frac{1}{\psi}\frac{\partial}{\partial\psi}\left[\psi D_{\psi\psi}\frac{\partial f_\perp(\psi)}{\partial\psi}\right]. \quad (4.12)$$

Similarly we can derive diffusion equation for parallel momentum. Since the maximum pitch angle is about $\psi_{\max}\approx 1/\gamma$, we can safely assume the boundary conditions, $f(p_\parallel,\psi\to\pi/2)=0$ [which is equivalent to assume $f(p_\parallel,p_\perp\to\infty)=0$]. Integrating Eq. (4.1) over pitch angle, we find the quasilinear diffusion equation for parallel momentum. In the small pitch angle approximation, the terms in the first pair of square bracket on the right-hand side are zero. The first term in the second pair of square bracket dominates, yielding

$$\frac{\partial}{\partial p_\parallel}\int D_{\parallel\psi}\frac{\partial f(p_\parallel,\psi)}{\partial\psi}\psi d\psi=-\frac{\partial}{\partial p_\parallel}\left[\frac{r_e}{\gamma}\hbar\omega_k n_k f_\parallel(p_\parallel)\right]. \quad (4.13)$$

The left-hand side of Eq. (4.1) is

$$\frac{\partial}{\partial p_\parallel}\int F_\parallel f(p_\parallel,\psi)\psi d\psi=-\alpha\frac{\partial}{\partial p_\parallel}[\gamma^2\langle\psi^2\rangle f_\parallel(p_\parallel)]. \quad (4.14)$$

Then, we find the one-dimensional quasilinear diffusion equation

$$\frac{\partial f_\parallel(p_\parallel)}{\partial t}=\frac{\partial}{\partial p_\parallel}\left\{\left[\alpha\psi_0^2\left(\frac{p_\parallel}{m_e c}\right)^2+\alpha_c\left(\frac{p_\parallel}{m_e c}\right)^4-r_e\left(\frac{m_e c}{p_\parallel}\right.\right.\right.$$
$$\left.\left.\left.-\frac{3}{4}\psi_0\frac{\omega_k}{\Omega_e}p_\parallel\frac{\partial}{\partial p_\parallel}\right)\hbar\omega_k n_k\right]f_\parallel(p_\parallel)\right\}, \quad (4.15)$$

where $\psi_0^2\equiv\langle\psi^2\rangle$. The first and second terms in the square brackets are the result of reaction force due to synchrotron and curvature radiation. The term $\propto n_k$ is diffusion due to waves, which increases the distribution tail (i.e., the acceleration effect). When this term is dominant with n_k being independent of $p_\parallel$, we have the solution $f_\parallel\sim p_\parallel$.

A. Spectral evolution

The evolution of the wave occupation number is described by

$$\frac{\partial n_k}{\partial t}+\frac{\partial}{\partial k}(\Gamma_{\rm M}kn_k)=-\Gamma_{\rm D}n_k, \quad (4.16)$$

where $\Gamma_{\rm M}$ is the growth rate of modulational instability, $\Gamma_{\rm D}$ is the damping rate due to NCR, and where we consider one-dimensional waves. For $\partial f/\partial t=0$ and $\partial n_k/\partial t=0$ we find

$$n_k\approx n_{k0}k^{-[1+(\Gamma_{\rm D}/\Gamma_{\rm M})]}. \quad (4.17)$$

Generally, we may assume $n_k=k^{-\xi}N(t)$. Then, from Eq. (4.16) we find

$$N(t)=N_0\exp\left[-\left(1+\frac{\Gamma_{\rm D}}{\Gamma_{\rm M}}-\xi\right)\Gamma_{\rm M}t\right], \quad (4.18)$$

where $N\equiv N_0$ at $t=0$. There are three possibilities that need to be considered separately. First, for $\xi=1+\Gamma_{\rm D}/\Gamma_{\rm M}$, we have $N(t)=$const corresponding to the solution given by Eq. (4.17). Second, for $|\xi|>1+\Gamma_{\rm D}/\Gamma_{\rm M}$ we have monotonically increasing $N(t)$. Finally, for $|\xi|<1+\Gamma_{\rm D}/\Gamma_{\rm M}$ we have an exponential decay spectrum with $n_k\to 0$ as $t\to\infty$.

B. Diffusion in pitch angles

If RQD can develop fully, we have $\partial f_{\parallel,\perp}/\partial t=0$, and since $D_{\psi\psi}$ is not sensitive to ψ (cf. Eq. 4.4), the evolved distribution is derived as

$$f_\perp(\psi)\approx A_1\exp\left(-\frac{\psi^2}{\psi_c^2}\right), \tag{4.19}$$

where $\psi_c=[2p_\parallel D_{\psi\psi}/(\alpha+\beta_R\gamma)]^{1/2}=[\hbar\Omega_e n_k r_e/(\alpha+\beta_R\gamma)\gamma^3]^{1/2}$ is the characteristic pitch angle caused by diffusion, and A_1 is a constant that is independent of ψ but in general may depend on $p_\parallel$. Similar to NQD, the RQD process can result in an increase in the particle's pitch angle. Through diffusion in the pitch angle, particles initially in the ground state can emit synchrotron/cyclotron radiation.

C. Diffusion in parallel momentum

We consider the case in which either n_k is small or γ is large. Then, Eq. (4.15) can be solved to yield

$$f_\parallel(p_\parallel)\approx A_2\left(\frac{m_ec}{p_\parallel}\right)^2\left[\alpha\psi_0^2+\alpha_c\left(\frac{p_\parallel}{m_ec}\right)^2\right]^{-1}, \tag{4.20}$$

where A_2 is a constant, which is independent of $p_\parallel$ and the n_k term is neglected. For very large γ but $\gamma<\sqrt{3\alpha/2}\psi_0R_B/e=\psi_0R_B\Omega_e/c$, deceleration due to cyclotron damping is important, and then, we have a cutoff

$$f_\parallel(p_\parallel)\approx\frac{A_2}{\alpha\psi_0^2}\left(\frac{m_ec}{p_\parallel}\right)^2. \tag{4.21}$$

For $\gamma>\psi_0R_B\Omega_e/c$, curvature radiation becomes dominant and the distribution has a much steeper cutoff, given by

$$f_\parallel\sim\left(\frac{m_ec}{p_\parallel}\right)^4, \tag{4.22}$$

which is much steeper than the cutoff due to synchrotron radiation.

V. CONCLUSIONS AND DISCUSSION

We have considered quasilinear diffusion arising from modulational instability of the low-frequency *L-O* mode by high-frequency transverse waves in the pulsar plasma. Owing to the modulational instability it is possible to convert the energy of the superluminal low-frequency *L-O* mode to low-frequency perturbations (with frequency $\omega^t-\omega'^t$). The unstable waves evolve into the subluminal region and can be absorbed through various resonances. We consider particularly NCR, as it is the most efficient absorption process. Cyclotron absorption can occur for particles in the energetic beam for the *O* mode with a small propagation angle. As a result of nonlinear interaction, the beam, which initially has very small spread, can develop a high-energy tail with nonzero pitch angles. These particles can emit synchrotron radiation, contributing to pulsar γ-ray emission [12,16]. There is a cutoff at high energy, which is determined by the deceleration due to synchrotron or curvature radiation being balanced by acceleration due to NCR damping.

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